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Mathematics

1. If $f(x) = x+7$ and $g(x) = x-7, x \in R$, find $(fog)(7)$
2. Evaluate : $\sin\left[\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right]$
3. Find the value of x and y if : $2\begin{bmatrix}1 & 3\\ 0 & x\end{bmatrix}+\begin{bmatrix}y & 0\\ 1 & 2\end{bmatrix}=\begin{bmatrix}5 & 6\\ 1 & 8\end{bmatrix}$
4. Evaluate: $\begin{vmatrix} a+ib & c+id \\ -c+id & a-ib \end{vmatrix}$
5. Find the cofactor of a_{12} in the following: $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$
6. Evaluate: $\int \frac{x^2}{1+x^3}\,dx$
7. Evaluate: $\int_0^1 \frac{dx}{1+x^2}$
8. Find a unit vector in the direction of $\vec{a} = 3\hat{i} - 2\hat{j} + 6\hat{k}$
9. Find the angle between the vectors $\vec{a} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + \hat{j} - \hat{k}$
10. For what value of λ are the vectors $\vec{a} = 2\hat{i} + \lambda\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + 3\hat{k}$ perpendicular to each other?

11. (*i*) Is the binary operation defined on set N, given by $a * b = \frac{a+b}{2}$ for all $a, b \in N$, commutative?

(*ii*) Is the above binary operation associative?

12. Prove the following:

$$\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{8} = \frac{\pi}{4}$$

13. Let $A = \begin{bmatrix} 3 & 2 & 5 \\ 4 & 1 & 3 \\ 0 & 6 & 7 \end{bmatrix}$.

Express A as sum of two matrices such that one is symmetric and the other is skew symmetric.

14. For what value of k is the following function continuous at $x = 2$?

$$f(x) = \begin{cases} 2x+1 & ; x < 2 \\ k & ; x = 2 \\ 3x-1 & ; x > 2 \end{cases}$$

15. Differentiate the following with respect to x : $\tan^{-1}\left(\frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}}\right)$

16. Find the equation of tangent to the curve $x = \sin 3t$, $y = \cos 2t$ at $t = \frac{\pi}{4}$

17. Evaluate: $\int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x}\, dx$

18. Solve the following differential equation:

$$(x^2 - y^2)\, dx + 2xy\, dy = 0$$

given that $y = 1$ when $x = 1$

19. Solve the following differential equation : $\cos^2 x \frac{dy}{dx} + y = \tan x$

20. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{j} - \hat{k}$, find a vector $\vec{c}$ such that $\vec{a} \times \vec{c} = \vec{b}$ and $\vec{a} \cdot \vec{c} = 3$.

OR

If $\vec{a} + \vec{b} + \vec{c} = 0$ and $|\vec{a}| = 3, |\vec{b}| = 5$ and $|\vec{c}| = 7$, show that the angle between $\vec{a}$ and $\vec{b}$ is $60°$.

21. Find the shortest distance between the following lines :

$$\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1} \text{ and } \frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$$

OR

Find the point on the line $\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2}$ at a distance $3\sqrt{2}$ from the point (1, 2, 3).

22. A pair of dice is thrown 4 times. If getting a doublet is considered a success, find the probability distribution of number of successes.

23. Using properties of determinants, prove the following :

$$\begin{vmatrix} \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \\ \beta+\gamma & \gamma+\alpha & \alpha+\beta \end{vmatrix} = (\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)(\alpha+\beta+\gamma)$$

$$\begin{vmatrix} \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \\ \beta+\gamma & \gamma+\alpha & \alpha+\beta \end{vmatrix} = (\alpha+\beta+\gamma)\begin{vmatrix} \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \\ 1 & 1 & 1 \end{vmatrix}$$

24. Show that the rectangle of maximum area that can be inscribed in a circle is a square.

25. Using integration find the area of the region bounded by the parabola $y^2 = 4x$ and the circle $4x^2 + 4y^2 = 9$.

26. Evaluate: $\int_{-a}^{a} \sqrt{\frac{a-x}{a+x}}dx$

27. Find the equation of the plane passing through the point $(-1, -1, 2)$ and perpendicular to each of the following planes:

$2x + 3y - 3z = 2$ and $5x - 4y + z = 6$

28. A factory owner purchases two types of machines, *A* and *B* for his factory. The requirements and the limitations for the machines are as follows :

Machine	Area occupied	Labour force	Daily output (in units)
A	1000 m^2	12 men	60
B	1200 m^2	8 men	40

He has maximum area of 9000 m^2 available, and 72 skilled labourers who can operate both the machines. How many machines of each type should he buy to maximise the daily output?

29. An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probability of an accident involving a scooter, a car and a truck are 0.01, 0.03 and 0.15 respectively. One of the insured persons meets with an accident. What is the probability that he is a scooter driver.

30. if:R→R where f(x)=4x+3 find f^{-1}

31. if$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 3 & 1 \\ 2 & 5 \end{pmatrix} = \begin{pmatrix} 7 & 11 \\ k & 23 \end{pmatrix}$ then write the value of k.

32. write the adjoint of matrix $\begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}$

33. find the value of $sin^{-1}(sin\frac{4\pi}{5})$,

34. evaluate $\int \frac{\log x}{x}\,dx$

35. if a line has direction ratios <2,-1,-2>,then what are its direction cosines?

36. Let R be the set of real numbers. if f:R→R is given byf(x) =4x+5,x ϵ R,and g:R→R is given by g(x)=$\frac{x}{x^2+1}$.find fog and gof,

37. prove that $cos^{-1}\frac{4}{5} + cos^{-1}\frac{12}{13} = cos^{-1}\frac{33}{65}$.

38. using properties of determinants prove that

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$$

39. find the relationship between 'a' and 'b' so that function (f) defined by

F(x)=$\begin{cases} ax+1 & if\ x \le 3 \\ bx+3 & if\ x \ge 3 \end{cases}$ is continuous at x=3.

40. differentiate $x^{x\ cosx} + \frac{x^2+1}{x^2-1}$.

41. Find the equation of the normal to the curve $y=x^3+2x+6$ which are parallel to the line x+14y+4=0.

42. Verify Longrange's mean value theorem for the function

F(x)=$\sqrt{(x^2-4)}$ in the interval[2,4]

43. Evaluate $\int \frac{2-3\sin x}{cos^2 x}\ dx$

44. solve the differential equation $(1+y^2)(1+\log x)dx + x\,dy = 0$

45. Show that $y=ae^{2x} + be^{-x}$ is a solution of the differential equation $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0$

46. if $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}, \vec{b} = \widehat{4\imath} - \widehat{2\jmath} + \widehat{3k}, \vec{c} = \hat{\imath} - 2\hat{\jmath} + \hat{k}$ find a vector of magnitude 6 units which is parallel to the vector $2\vec{a}$- $\vec{b}$+ $\overrightarrow{3c}$.

47. Find the shortest distance between the lines whose vector equation are

$\vec{r} = (1-t)\hat{\imath} + (t-2)\hat{\jmath} + (3-2t)\hat{k}$

$\vec{r} = (s-1)\hat{\imath} + (2s-1)\hat{\jmath} + (2s+1)\hat{k}$.

48. Five dices are thrown simultaneously. If the occurrence of a pair of dice is thrown 5 times .if getting a doublet is considered a success, find the probability of two successes.

49. Find the vector equation of a line passing through the point (1,2,-4) and perpendicular to the lines $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}, \frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$.

50. Using matrices ,solve the following system of equations

X+2y-3z=-4, 2x+3y+2z=2, 3x-3y-4z=11.